

Application of Theory to Propeller Design¹

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Subcavitating propeller design theory has been developed to the extent that theoretically designed propellers perform as predicted. Such design procedures depend heavily on the availability of high-speed digital computers for making lifting-line and lifting-surface calculations. In spite of these developments, many aspects of propellers remain an art.

Supercavitating propeller design theory has also been developed, but the computer programs necessary for application are still under development. Of necessity, the design of these propellers follows semiempirical procedures.

The various theories available to the designer are reviewed. Emphasis is given to the key design decisions which face the designer.

INTRODUCTION

Purpose of the Paper

During recent years, the availability of high-speed digital computers has provided the stimulus for the formulation of improved propeller design procedures. Much more adequate mathematical models can be used to represent the hydrodynamic action of a marine propeller than was hitherto possible. A further outcome of using a computer is that the designer, released from the drudgery of performing laborious computations, is better able to exercise critical design judgments and to practice his "art." The major purpose of this paper is to emphasize the art of contemporary propeller design, as seen by the authors, and to discuss propeller theory and computational procedures mainly from the viewpoint of available techniques.

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Development of Subcavitating Design Procedures

For many years, the design of subcavitating propellers depended to a large extent on experimental series data and a number of extensive series have been developed (e.g., Taylor, Troost, and Gawn, refs. 1, 2, 3, and 4). These series are still very helpful for preliminary design studies, but they are seldom applied to a specific propeller design today, since the propeller must be constrained to a specific geometry. This geometry may be unsatisfactory for reasons of cavitation and/or vibration, particularly for propellers which operate in a wake.

More recently, most design methods depended on lifting-line theory of one form or another with finite-width effects being considered in an empirical, semiempirical, or approximate manner (e.g., Schoenherr, ref. 5; Lerbs, ref. 6; Hill, ref. 7; Van Manen, ref. 8; McCormick, ref. 9; and Eckhardt and Morgan, ref. 10). Two common "approximations" were use of Goldstein factors and camber correction factors, to allow for effects of blade number and aspect ratio, which were not strictly applicable to the propeller being designed. These approximations were necessary before the advent of high-speed digital computers, due to the computational effort required by the more rigorous solutions. Today, with computers, both lifting-line and lifting-surface calculations can be performed in sufficient detail to make rigorous theoretical design calculations feasible and desirable.

Development of Supercavitating Propeller Design Procedures

Initial attempts at the David Taylor Model Basin (DTMB) to formulate a design method were made by Tachmindji and Morgan (ref. 11). This method essentially grafted two-dimensional supercavitating design theory data onto a subcavitating design method (ref. 10) then in current use at DTMB. This method used conventional subcavitating lifting-line theory (i.e., Goldstein factors) and involved the use of lifting-surface corrections which did not account for cavity interaction effects. Test results for models designed by this method are described by Hecker, Peck, and McDonald (ref. 12) who showed that these model propellers did not generally perform as predicted.

Posdunine (see reference 13 for a list of this Russian author's references) pointed out that the blade cavities affect propeller performance in ways other than that due to two-dimensional supercavitating hydrofoil characteristics. Cavity-blade interference and cavity thickness effects can lead to major differences between noncavitating and supercavitating propeller performance. Currently used design methods include approximate recognition of these effects. The Naval Ship Research and Development Center (NSRDC, formerly DTMB) makes use of current

subcavitating lifting-line and lifting-surface methods, with cavity-blade interference effects accounted for by superimposing an estimated cavity thickness onto the lifting-surface calculations. Hydronautics (ref. 13) also makes use of subcavitating propeller lifting-line theory and corrects for lifting-surface effects by use of camber correction factors (ref. 14). In addition, cavity-blade interference effects are approximated by use of data derived from a study of a two-dimensional supercavitating hydrofoil in proximity to a free surface. More recently, Titoff, Russetsky, and Georgiyevskaya (ref. 15) and Davis and English (ref. 16) described the problems associated with actual applications of their design methods. In this respect, a very detailed description of a supercavitating propeller strength investigation is given in reference 16.

Attempts are continuing to develop more adequate supercavitating propeller design methods based on a correct recognition of the effects of the blade cavities. English (ref. 17) has formulated a lifting-line supercavitating propeller theory, based on an extension of Goldstein's work for a subcavitating finite-bladed propeller (ref. 18), with modified boundary conditions to allow for the effect of the cavities. Cox (ref. 19) has devised lifting-line and lifting-surface theory for the determination of induced velocities (the central problem of theoretical design). The effect of the blade cavities is accounted for by use of pressure source singularities. Laurentiev (refs. 15 and 20) has also formulated a propeller lifting-surface solution based on a similar model. No numerical results or design calculations and model test results are available for these new theories.

Only limited experimental series data are available for supercavitating propellers (e.g., Newton and Rader, ref. 21, and Georgiyevskaya, ref. 22). One reason that more series data are not available is that a supercavitating propeller operates efficiently over only a small range of advance coefficient. At advance coefficients much higher than design, pressure-side cavitation occurs and the performance falls off and at advance coefficients much lower than design, the suction-side cavity becomes very large since the section is operating at a relatively high angle of attack and the performance also falls off. Thus, for a good performing supercavitating propeller series, it is not sufficient to have a simple variation of pitch but it is necessary to vary the blade-section shape for different advance coefficient ranges. This necessity makes obtaining data for a series of good performing supercavitating propellers very expensive. To furnish data for use in design, Caster (ref. 23) developed a theoretical "design-point" series for supercavitating propellers based on the method of Tachmindji and Morgan (ref. 11). The experimental series work of Newton and Rader (ref. 21) furnishes limited data but has its main application for a speed range where the cavitation may not be fully developed; i.e., 50 knots or less.

The Advantages and Disadvantages of Theoretical Design

The use of a design method based on a rational theory permits the designer a free choice with respect to the following features: (1) number of blades, (2) hub size, (3) radial blade loading, (4) chordwise blade loading, (5) blade shape and size, (6) blade skew, and (7) wake adaption. All of these features, with the possible exception of hub size, can play a major role in minimizing ship vibration and/or cavitation problems. The proper recognition of propeller blade strength requirements can only be conveniently satisfied within the framework of a theoretical design method, and the strong interdependency between efficiency and strength for a supercavitating design makes the use of a theoretical design method mandatory for most applications.

With the possible exception of the so-called optimum subcavitating propeller in uniform inflow, a high-speed digital computer is essential to perform the very laborious and complex computations associated with present-day lifting-line and lifting-surface theories for marine propellers. This is an obvious disadvantage since the programs and computing facilities may not be readily available. Also, even the most keen proponent of computerized lifting-line and lifting-surface computations has to be aware of a very serious drawback of this approach. Such design calculations are very prone to undetected numerical mistakes, and the detection and elimination of these mistakes requires great care in use and understanding of the computer programs.

For the case of subcavitating propeller design, the theoretical design procedures are generally quite adequate. However, in the case of supercavitating propeller design, the theoretical design procedures, mainly due to lack of computer programs, have not been adequately developed. At the same time, the more approximate approaches are not satisfactory even for power performance, whereas for the subcavitating propeller, these approximate approaches are often adequate for this purpose.

DESIGN CONSIDERATIONS

Phases of Design

Basically, there are five separate phases in the design of a propeller; namely

- (1) Preliminary design analysis to determine the design parameters for which the propeller is to be designed in order that the propeller be compatible with the ship, installed propulsion machinery, and transmission from the standpoint of efficiency and vibration. In addition, for supercavitating operation, it is necessary to ensure that the design

parameters are chosen to permit the blades to operate in a supercavitating regime.

(2) Determination of the desired radial load distribution, together with the radial hydrodynamic pitch angle; i.e., lifting-line calculations. The theory is for inviscid flow but the viscous drag can be taken into account suitably by a strip theory analysis. For the sake of convenience for supercavitating operation, blade cavity pressure drag is considered in association with blade viscous drag.

(3) Determination of the blade shape, area, and thickness distribution from the standpoint of cavitation and strength. For subcavitating propellers, cavitation erosion and thrust breakdown should not occur at the design conditions. For supercavitating propellers, blade pressure-side cavitation should not occur and the suction-side cavity should completely enclose the suction side of the blade at the design conditions.

(4) Determination of the final camber and pitch distributions from lifting-surface calculations.

(5) A strength check for the final configuration.

Methodical series data can play a useful role during preliminary design analysis; i.e., phase (1). The main emphasis of theoretical propeller design calculations is concerned with lifting-line and lifting-surface calculations; i.e., phases (2) and (4), respectively. Phase (3) is concerned with design of the propeller from the standpoint of both cavitation and strength for subcavitating propellers and both strength and efficiency for supercavitating propellers. Phase (5) makes use of detailed intermediate results determined during the design procedure.

Ship Propulsive Performance and Basic Propeller Design Considerations

As already mentioned, the propeller designer attempts to select design parameters to achieve effective ship propulsion characteristics; i.e., minimize delivered power P_D to the propellers for a given ship operating speed V or maximize ship operating speed for a given delivered power. These simple objectives often have to be compromised to some degree in order to meet constraints imposed by ship and appendage geometry, machinery and gearing, minimization of vibration and cavitation erosion, and, last but not least, ship construction costs. The propeller designer cannot usually exercise a choice regarding number of propellers, shaft inclination, maximum diameter, and shaft and hub size. He will nearly always have to meet the required propulsion characteristics within a narrow range of rpm.

The design conditions are characterized by the thrust loading coefficient C_{T_h} (or power coefficient C_P), and advance ratio λ . For the selected ship speed, their specification involves the thrust T the propeller has to deliver

(or power P to be absorbed), diameter D , propeller revolutions n , and speed of advance V_A . The determination of these parameters usually involves model basin tests of the appended hull,

$$C_{T_h} = \frac{T}{(\rho/2)\pi R^2 V_A^2} \quad C_{T_h} = \frac{T}{(\rho/2)\pi R^2 V^2} \quad (\text{wake adapted})$$

$$C_P = \frac{P}{(\rho/2)\pi R^2 V_A^3} \quad C_P = \frac{P}{(\rho/2)\pi R^2 V^3} \quad (\text{wake adapted})$$

$$\lambda = \frac{V_A}{\pi n D} \quad \lambda = \frac{V}{\pi n D} \quad (\text{wake adapted})$$

both with and without "stock" propellers to measure the resistance, thrust, torque, and rpm. From these measurements the effective horsepower P_E , delivered horsepower P_D , thrust deduction t , and effective wake w (derived from propeller open-water curves) and, hence, propulsive efficiency are determined approximately:

$$\eta_D = \frac{P_E}{P_D} = \eta_B \eta_H = \eta_B \frac{(1-t)}{(1-w)}$$

where η_H is hull efficiency and η_B is propeller behind efficiency.

Every towing tank has its own approach for correcting the various elements of the propulsive efficiency to allow for model/ship correlation. The important thing to note is that the thrust deduction factor and the wake fraction are dependent on the choice of the stock propeller, which is chosen to approximate the final propeller design with regard to diameter, pitch-to-diameter ratio P/D , blade area, and number of blades Z , diameter being the most important parameter. A different final choice for diameter, especially when the propeller operates substantially in a wake, requires a new estimate for the affected components.

In some cases, where self-propulsion tests are not available, it may be necessary to estimate the thrust deduction and effective wake. The work of Harvald (ref. 24) and Todd (ref. 2), and the work on the Series 60 (refs. 25 and 26) can be used for making this estimate. Also, Beveridge (ref. 27) has shown that it may be possible to calculate the thrust deduction from theoretical considerations.

During this stage of design, the velocity field in the plane of the propeller is determined by making wake measurements on the model to determine the inflow velocities to the propeller. If wake measurements are not made, an estimate of these wake velocities must be made. The work of Hadler and Cheng (ref. 28) can be used for estimation purposes. From these measurements, *not only* is the spatial flow pattern obtained, but also the circumferential average of the inflow velocities. The circumferential

averaged velocities are a necessary input to the propeller design so that the propeller can be designed for the desired radial load distribution. The so-called "optimum" load distribution (Troost, ref. 29, and Morgan and Wrench, ref. 30) can usually be specified unless other considerations such as cavitation and vibration are important. Many propellers have been designed for load distributions radically different from "optimum" without a significant loss in propeller efficiency. This is due to the fact that the optimum load distribution is usually only concerned with minimum induced drag. If a load distribution is specified, it should ensure that maximum blade load is carried in the vicinity of midblade. Placing maximum load toward the extreme blade root or tip can result in an appreciable loss of efficiency.

Subcavitating propeller efficiency is not strongly dependent on blade number, provided that flow choking does not occur near the hub. If it is necessary to consider a large number of blades, then the hub-to-propeller diameter ratio must be increased. Experience has shown that it is quite safe to consider five blades with a hub-to-propeller diameter ratio of 0.2, except for very low pitch ratios. Supercavitating propeller efficiency is strongly dependent on the number of blades. Off-design efficiency of these propellers can deteriorate due to the proximity of a blade cavity to the pressure side of a neighboring blade. The greater the blade number, the greater is this effect.

It is at this stage that the decision is made whether the design should be for a subcavitating or a supercavitating propeller. The decision is based on the local cavitation number of the blade sections. Criteria (ref. 11) indicate that the design should be a supercavitating one if the local cavitation number at 0.7 radius, $\sigma_{0.7}$, is less than 0.05, and a subcavitating one if $\sigma_{0.7}$ is greater than 0.1. Figure 1 presents the desired operating regions for subcavitating and supercavitating propellers in terms of cavitation number σ and advance coefficient J . Lines 1 and 2 indicate $\sigma_{0.7}$ values of 0.1 and 0.05, respectively. If the operating region falls between Lines 1 and 2, the decision concerning type of design will depend on the application. It should be pointed out the use of cavity "ventilation" can effectively reduce the local cavitation number of the blade sections, which can mean that an operating condition originally between Lines 1 and 2 can be moved below Line 2.

Vibration

Propeller-associated vibration comes from two sources: (1) the unsteady bearing forces which are associated with the propeller blades operating in a spatially varying wake field and unsteady forces being introduced through the shaft and (2) the pressure forces which are associated with the rotation of the propeller past the hull. Both can be controlled to some extent by the propeller design. To minimize unsteady

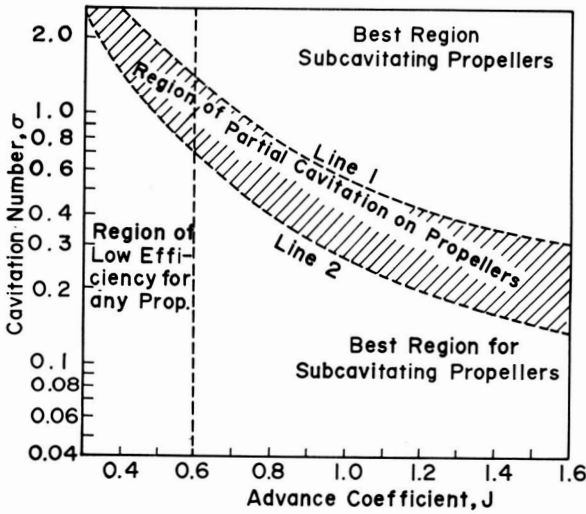


FIGURE 1.—Propeller design ranges from considerations of cavitation.

bearing forces, it is necessary to avoid coincidence of blade number, or blade number ± 1 , with strong wake harmonics (ref. 28). A Fourier series analysis of the spatially varying flow into the propeller will provide excellent guidance in the selection of the number of blades. Also, in the selection of number of blades, it is essential to avoid exciting a main hull resonance at normal operating speeds. By changing the blade number, it is often possible to alter the blade-rate excitation of hull resonance to an unimportant speed.

The propeller pressure forces are dependent on the clearances between the ship and the propeller, the blade thickness, and the blade loading. The clearances should be selected with care. If the clearances are too small, vibration will be a problem, and if too large, arrangement problems may arise. A number of criteria have been developed to help select the

TABLE I.—Effect of Thickness Distribution on Section Modulus for Typical Airfoil Sections

Section	C_s
NACA 16 series.....	0.0890
NACA 65-A series.....	0.0775
NACA 66 series.....	0.0848
TMB-EPH section.....	0.0899

clearance (refs. 31, 32, 33, and 34). The proper clearance will usually be the factor which controls the propeller diameter.

Blade rake and skew may be used to help minimize vibration. Blade rake (i.e., angular displacement of the blade centerline in the propeller axis plane) is often used for propellers which work in an aperture to increase the distance between the blades and the hull and appendages forward of the propeller. Blade skew (i.e., differential displacement of chords along helical reference lines) permits a more gradual entry of the blade into a high wake region. Skew also increases clearance between the blades and the hull and appendages forward of the propeller.

Strength

A propeller blade must contain enough material to keep the stresses within the blade below a certain predetermined level. This level depends on the material properties with regard to both steady-state and fatigue strength and both mean and unsteady blade loading. The material selection controls the allowable stress level and the blade chord and thickness are the main parameters which control this stress level. The maximum stress for subcavitating propellers normally occurs at the point of maximum blade thickness unless the blade is unusually distorted. Without consideration of the centrifugal forces and blade rake and skew, the required maximum blade thickness-chord ratio, t/c , for subcavitating propellers, can be approximated by the following equation:

$$\frac{t}{c} = \left(\frac{M_{x_0}}{C_s C_c c^3 \sigma_a} \right)^{1/2}$$

where M_{x_0} is the bending moment about the blade chord (obtained from lifting-line calculations), σ_a is the allowable blade stress, C_s is a coefficient dependent on the chordwise thickness distribution (given in table I), C_c is a coefficient dependent on the chordwise camber distribution, and c is the section chord. The coefficients C_s and C_c were derived from Milam and Morgan (ref. 35) by Barr (ref. 36). McCarthy and Brock (ref. 37) in their measurements of propeller stress indicate that the principal axes follow almost along the section meanline. This means that for subcavitating propellers C_c should be assumed equal to 1.

The maximum thickness for supercavitating propellers cannot be derived on such a simple basis (refs. 37 and 38). In fact, the design of these propellers from the standpoint of stress is unsatisfactory.

All propellers should be designed considering both mean and unsteady loads acting on the blade. These must be considered together since the allowable unsteady stress decreases as mean stress increases. A simple equation for the allowable stress level can be derived from the Goodman

diagram (ref. 39) for both the mean and unsteady load. This equation is known as the equation for the Goodman safe region

$$\sigma_f = \frac{\sigma_{\max}(2 - R_s)}{3}$$

where

R_s ratio of minimum stress to maximum stress during a cycle

σ_f fatigue stress for a given number of cycles, single amplitude (table II)

σ_{\max} maximum stress for a given number of cycles.

In principle, the unsteady forces on a blade can be calculated theoretically (Tsakonas, Breslin, and Miller, ref. 40). The available theoretical procedures are not yet in a form which can be used easily; as an approximation, the method of McCarthy (ref. 41) can be used for unsteady loading at the blade root.

The calculation for t/c is made at each radius to obtain the blade thickness distribution. For practical application, certain limitations are usually applied for minimum thickness and judicious fairing is used. For instance, the thickness at the tip, for fairing purposes, is often assumed to be 0.003 times the propeller diameter. From the standpoint of stress, it is always better to have the blade slightly too thick than too thin.

The simple formulation presented is for the initial check on stress. Once the design is completed and the geometry of the propeller is known, a strength check which includes the effect of centrifugal force and blade rake and skew must be made. Unfortunately, the theory is not adequately developed to make what would be considered to be a satisfactory calculation. McCarthy and Brock (ref. 37) have shown that simple beam theory gives a good approximation to the maximum stress for a wide-bladed, unskewed, subcavitating propeller, but Boswell (ref. 42) has

TABLE II.—*Fatigue Strengths (psi) in Seawater, 10⁸ Cycles*

Material	σ_a
Manganese bronze.....	6 000
Nickel-manganese bronze.....	6 000
Nickel-aluminum bronze (Ni-Bral).....	12 500
Manganese-nickel-aluminum bronze (Superston 40—Grade 5).....	9 000
CF-4 stainless steel.....	6 500
	(20 000 with cathodic protection)

shown that it is not adequate for blades with large amounts of skew. McCarthy and Brock also conclude that present procedures are not adequate for determining the stress in a supercavitating propeller. Similar results have also been shown by Davis and English (ref. 16).

Cavitation

The designer must try to ensure that a propeller designed for sub-cavitating operation does not suffer from thrust breakdown and is not prone to erosion due to cavitation (Morgan and Lichtman, ref. 43, and Newton, ref. 44). Initially, during the preliminary design stages, the minimum expanded area ratio (EAR) should be determined to ensure freedom from thrust breakdown for the maximum power condition. The Burrill chart (ref. 45), or equivalent (ref. 46), is probably sufficient for this purpose.

The detailed procedure for performing a check to ensure freedom from blade cavitation is carried out after the lifting-line calculations. This check relies on the results of experimentally observed cavitation inception data (ref. 47), or theoretically predicted data (refs. 48, 49, and 50) for sections of prescribed thickness to chord ratio t/c , and meanline cambered for a design lift coefficient C_{L_i} . Figure 2 shows an example of such data for a range of angles of attack against local cavitation number σ_r , with t/c as a parameter. The insides of the "buckets" are cavitation-free regions with the top and bottom of the buckets indicating leading-edge suction-side (back) and pressure-side (face) cavitation, respectively. A study of these curves will immediately reveal a tradeoff choice; namely, that by selection of t/c it is often possible to achieve increased latitude against leading-edge

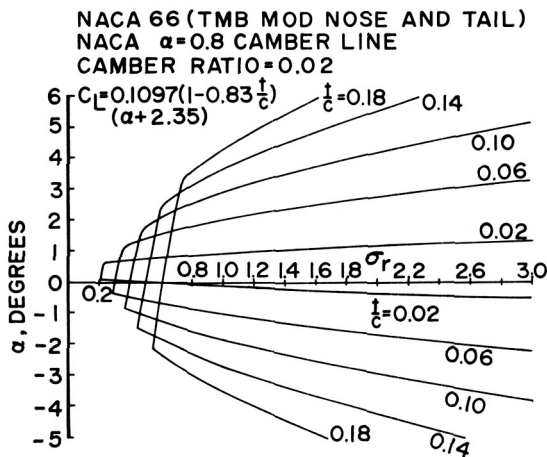


FIGURE 2.—Typical cavitation inception diagram for a two-dimensional section.

cavitation due to fluctuating angle of attack at the price of earlier onset of back bubble cavitation at shock-free entry. The type of information shown in figure 2 can easily be derived for any desired meanline and basic thickness form by the method of linear superposition (ref. 51). Such data can easily be condensed into a more convenient form for design use (refs. 49 and 50).

Figure 3 shows the wake for one radius value at the propeller position for a typical single-screw merchant ship. At 0.8 radius, the flow angle varies about the mean from -8 degrees to $+2$ degrees during each revolution. Hence, for a propeller which has to operate in a wake, it is necessary to obtain measurements or a good estimation of the inflow into the propeller. Both circumferentially averaged flow angles $\beta(r)$ and maximum positive and negative variations in flow angle about the mean are required for the operating condition under investigation. In order to obtain an engineering estimate for the effective variations in flow angle, an adaptation of the method described by Lerbs and Rader (ref. 52) can be used. This makes use of the concept of "effective aspect ratio" for each section of a propeller blade. The wake shown in figure 3 would result in an effective angle of attack variation of somewhat less than $+8$ degrees to -2 degrees.

If, in the use of the cavitation diagrams, the thickness ratio is smaller than required for stress and the cavitation diagrams indicate that thicker

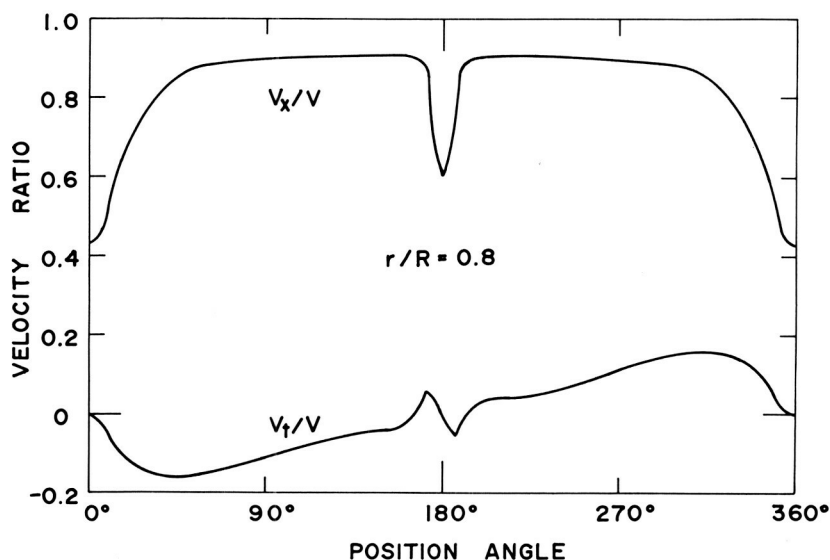


FIGURE 3.—Representative velocity distribution in plane of propeller for single-screw ship.

sections will cavitate, the section chords should be increased and the calculations repeated until a balance is achieved between cavitation and strength criteria.

For the uncommon situation where cavitation is absolutely no problem, the section chord and thickness are chosen for minimum drag and to meet the strength criteria. For frictional drag, the chord should be as short as possible, but if the thickness-chord ratio is too large, separation will occur on the foil and the form drag will increase. Although a precise maximum value of the thickness-chord ratio cannot be given, it is probably best to limit the maximum value to 0.18 to 0.22. This value may also depend on the lift of the foil.

A common situation is one in which the angle of attack variation is so great and/or the cavitation number is so low that it is impossible to pick a section shape which will not cavitate. If the cavitation number is sufficiently low, then a fully cavitating propeller should be designed. In those instances where this is not possible, and where cavitation arises mainly from an angle of attack variation, it is desirable to use the cavitation diagrams to minimize the occurrence of cavitation. How a particular subcavitating propeller which falls into this category is designed depends on the particular ship's operating characteristics. The cavitation and strength calculations are usually carried out at the maximum power condition. If cavitation is unavoidable at this condition and the ship operates generally at some lower speed, say a cruise speed, then the selection of blade chord should be based on the cruise condition or some higher speed if possible. The strength is always based on the maximum power condition. For most ships, there is little change in the thrust and power coefficient between cruise and maximum speed; therefore, this speed change can be accounted for by multiplying the cruise cavitation number by the ratio of the two speeds squared (for the lower speed the cavitation number will be higher). Thus, in essence, the blade thickness-chord ratio is selected to give the maximum cavitation-free speed. Care must be exercised that sufficient blade area is provided to prevent thrust breakdown at the maximum speed. Model tests, of course, should be conducted to help ensure that cavitation erosion will not occur. In this regard, any bubble cavitation on the blade should be avoided.

In general, the approach outlined when the propeller is operating in a wake field will result in sections with higher thickness-chord ratios than if the propeller is designed as if it were operating in a steady flow. The thicker sections are less susceptible to cavitation caused by angle of attack variations but are more susceptible to cavitation if no angle of attack variation occurs.

It will not be necessary to repeat the lifting-line calculations after the thickness and cavitation calculations are made unless large changes are made in the chord ratio. This will influence the section viscous drag and

the importance of this drag is dependent on the lift coefficient and the propeller hydrodynamic pitch.

In addition to blade area and thickness, the designer possesses considerable freedom of choice in number of blades, blade shape, and radial and chordwise load distribution to minimize the occurrence of cavitation. None of these parameters, within reasonable limits, has a strong effect on propeller efficiency for a subcavitating propeller, and a considerable choice is offered to delay cavitation.

If it is considered necessary to avoid the impingement of tip and hub vortex cavitation on rudders, it is possible to delay this type of cavitation by unloading the blade tip and root. As already pointed out, nonoptimum radial load distributions have very little effect on efficiency.

Any selected blade outline should have a shape without kinks or points of inflection such that its rate of change with respect to radial position is smooth and moderate. Experience has indicated that a kink in the leading-edge outline is often the origin of cavitation. It is good design practice for a subcavitating propeller to have the maximum chord position in the vicinity of maximum blade loading. Root chord should be selected on the basis of number of blades and amount of flow inclination due to the stern shape and shaft angle, bearing in mind the danger of root cavitation. For a supercavitating propeller blade, it is usual to select a blade shape which has maximum chord near the root in an attempt to maintain effective blade strength. A comprehensive family of blade shapes is given by Cox (ref. 14) who defines formulae for blade size and shape in terms of number of blades, expanded area ratio, position of maximum chord and ratio of root chord to maximum chord.

The choice of chordwise load distribution should be based on avoidance of cavitation in the case of a subcavitating design and minimizing induced cavity drag in the case of a supercavitating design. In the past, designers often favored constant chordwise loading, which corresponds to the $NACA(a=1.0)$ two-dimensional meanline loading. Due to viscous effects, such loading cannot be sustained toward the trailing edge. This has been demonstrated by two-dimensional foil tests which indicated that the $NACA(a=1.0)$ meanline realized only 74 percent of its theoretical lift (ref. 51). Since theoretical propeller design methods can only account for viscous effects by use of simple strip-theory methods, the use of a theoretical design procedure with some form of semiempirical corrections is necessary when a constant load meanline is used. If one wishes to design a propeller by a completely theoretical procedure, it is necessary to use a chordwise load distribution for which there is good agreement between potential flow and experimental lift, at least in two dimensions. An excellent choice, backed by experimental two-dimensional and propeller model tests, is that equivalent to a $NACA(a=0.8)$ meanline loading; i.e., constant load from the leading edge to 80-percent chord,

followed by a linear decrease to zero at the trailing edge. Such type of loading, of course, involves an ideal angle of attack.

Another type of chordwise loading which may offer considerable advantages in the case of avoidance of leading-edge cavitation due to fluctuations in flow angle from operating in a wake is a "roof-top" type loading; i.e., symmetrical about midchord and increasing linearly from zero at the leading edge to a certain percent of chord, and thence constant to some other percent of chord, followed by a linear decrease to zero at the trailing edge (refs. 49, 53, and 54). Such a loading has not been investigated experimentally.

Interaction Between Strength and Performance in a Supercavitating Propeller

The difficulty in the design of supercavitating propellers, besides cavity interference, is the interaction which occurs between performance and strength. For the subcavitating propeller, there is essentially no such interaction, as the section drag coefficient is independent of the blade thickness for the sections usually used in marine propellers. For supercavitating propellers, there is an inverse relationship between efficiency and strength. The blade material is placed between the camberline and the cavity on the suction side. For low cavity drag, the blade section should be highly cambered and have a thin cavity which means a thin, highly stressed blade. To decrease the blade stress, the blade must be thicker, especially at the leading edge. To accommodate this increased thickness the cavity is made thicker by decreasing the camber and increasing the angle of attack. A considerable increase in the section drag will result and the propeller will be less efficient.

The design calculations not only involve the approximation of the induced velocities due to both loading and thickness, but an iteration must be carried out to obtain the maximum efficiency at the safe stress level in the blade. Coefficients for the section modulus have been derived for a number of sections (Morgan, ref. 38), and the change in cavity drag can be approximated theoretically (ref. 13). How the cavity drag is affected by the three-dimensionality of the flow is not known.

During the design of the supercavitating propeller, not only are the camber and angle of attack varied but also the section chord. This will have an effect on both the drag and the strength. Because of the sharp leading edge generally necessary for low-drag supercavitating sections, the sections cannot take a negative angle of attack during a revolution of the blade without severe pressure-side cavitation occurring at the leading edge. To offset this cavitation problem, it is often necessary to design the section for a higher angle of attack than for minimum drag or strength. Thus, the requirement of operating in a wake field will often have an

adverse effect on the performance. During the design of supercavitating propellers, it is necessary to examine closely the wake field in which the propeller must operate and ensure, if necessary by redesign of the hull or appendage system in front of the propeller, that the propeller sections do not operate at negative angles of attack.

Another way the stress of fully-cavitating propellers has been decreased is by adding a thickness distribution to the section, usually parabolic (ref. 55). This can give a lower stress level with less drag penalty than using angle of attack. However, it does not improve the section performance when there is a varying angle of attack.

THE FOUNDATIONS OF THEORETICAL PROPELLER DESIGN

The distinct and important roles played by lifting-line and lifting-surface theory have already been pointed out. Satisfactory numerical procedures now exist for inclusion in subcavitating propeller design methods. Unfortunately, this is not yet the case for supercavitating propeller design methods. The historical development of these theories and identification of the significant contributions in this field was reported during 1969 by Cox (ref. 56).

Propeller Lifting-Line Theory

Propeller lifting-line theory follows from Prandtl's mathematical treatment of the lifting-line theory as applied to wings of finite length. This model cannot account for chordwise effects but can recognize spanwise (radial) effects. Thus, now that it has been extensively developed for moderately loaded propellers, it provides a very satisfactory and powerful tool.

The lifting-line theory of propellers is described as follows. It is assumed that each propeller blade can be replaced by a lifting line and that the circulation varies along the propeller radius. From vortex theory, it follows that free vortices are shed from the lifting line and, in a coordinate system which rotates with the propeller, these free vortices form a general helical surface behind the propeller. Since the free vortices follow streamlines in the rotating coordinate system, the pitch of the helical vortex sheet is taken to be dependent on the free-stream velocity, the speed of rotation of the propeller, and the velocities induced by the vortex system. For the supercavitating propeller, the pitch of the helical vortex sheet should also probably depend on the velocities induced by the blade and cavity thickness. In principle, the effect of thickness could be included in the lifting-line theory of subcavitating propellers, but the magnitude of

these induced velocities from thickness can generally be ignored in this phase of the calculation, although thickness effects must be included in making lifting-surface corrections. For propellers with large numbers of blades and low blade pitches, the induced velocities from the blade thickness should be included in the lifting-line calculations.

Although the lifting-line is a very simple representation of a propeller blade, the theory has proven very useful for the design of propellers. From a combination of lifting-line theory, the Kutta-Joukowski law, and strip theory, it is possible to determine the radial distribution of hydrodynamic pitch and load which will produce the required propeller thrust and torque. In addition, detailed calculations can be made regarding efficiency, cavitation, and strength. These calculations all require the use of iteration procedures but are much simpler than performing the lifting-surface calculations.

A discussion of the details of subcavitating lifting-line theory and the associated computational procedures was given in 1965 by Morgan and Wrench (ref. 30). This work includes the significant contributions of Betz (1919), Goldstein (1929), Moriya (1933, 1942), Kawada (1936), Strscheletzky (1950), and Lerbs (1952). The lifting-line procedure of Lerbs is used in the NSRDC design procedure. No satisfactory procedure is yet available for supercavitating propellers. A recent study was presented by Cox in 1968 (ref. 19).

The main hypotheses and assumptions of the lifting-line concept are summarized as follows:

(1) The fluid is inviscid and incompressible. However, in the calculation procedure, allowance is made by the use of strip theory for the viscous drag of the blade for subcavitating propellers and viscous and cavity drags for supercavitating propellers.

(2) The free-stream velocity is axisymmetric and steady. Consequently, the propeller may be wake-adapted.

(3) Each propeller blade is replaced by a lifting line and the circulation varies along the radius.

(4) Each of the free vortices is at a constant pitch in the downstream direction, but a radial variation in pitch is allowed. This means that effects of slipstream contraction and centrifugal force on the shape of the vortex sheets are ignored.

(5) The radial velocity induced at the hub is assumed small so that the effect of the hub on the pitch of the trailing sheets is ignored. For very large hub propellers, this assumption would not be valid.

(6) For the supercavitating propeller, a pressure-source representation on the trailing vortex sheet can be used to represent the cavity thickness. The strength and length of the source distribution will then depend on the circulation and the free-stream cavitation number.

An additional assumption made is that the circulation is zero at the propeller hub. It is obvious that the circulation must go to zero at the blade tip but, at the blade hub, the assumption is justified on the basis of an argument by Wald (ref. 57). He showed that for a hub of finite length behind the propeller the circulation must go to zero at the hub for an optimum propeller and it can be inferred from this work that the circulation must always be zero at the hub for a hub of finite length. On the other hand, if the hub is of infinite length, the circulation should have a finite value at the hub for an optimum distribution (McCormick, ref. 58). Since most propellers will not be optimum and because of Wald's results, it seems reasonable to assume that the circulation will be zero at the propeller hub.

For propeller design, it is necessary only to calculate the velocity induced by the vortex system at one of the bound vortex lines. Only the axial and tangential components of this induced velocity are considered, as the radial component does not contribute to the forces on the propeller.

Propeller Lifting-Surface Theory

Lifting-line theory cannot recognize chordwise effects, and thus provides no information about how to achieve the required radial distribution of lift for the relatively broad-bladed marine propeller. It is not sufficient to insert cambered sections, of known chord and lift coefficient in two-dimensional flow, at the requisite radial stations, since they will develop much less lift once they are part of a low-aspect ratio propeller blade. In order to determine section camber, it is necessary to use a more adequate vortex theory model for the propeller blades which recognizes both chordwise and radial distribution of lift. Lifting-surface theory can be used to achieve this purpose since the known (chordwise integrated) radial circulation distribution predicted by lifting-line theory can be distributed in any desired manner in the chordwise direction. In addition, the radial pitch distribution of the lifting surface is known from the lifting-line theory. In principle, it is then possible to calculate the shape and position of the cambered meanline surface with reference to the lifting-surface position, such that the required radial and chordwise lift distribution is achieved. (Recognizing the effects of blade skew, if necessary, adds no complication to a lifting-surface procedure.)

The assumptions which have been put forward previously for lifting-line theory hold for lifting-surface theory except that now it is assumed that the bound vortices are distributed over the blade surface rather than concentrated at a lifting line and the blade is allowed to have a finite thickness. From the standpoint of the linearized theory, the loading and thickness effect can be considered separately. Also, the theory is approached from the standpoint that it is required to design a propeller to

produce a certain thrust, or power, at a specified speed coefficient and not from the standpoint of determining the performance of a given propeller.

Another consequence of linearized theory is the assumption that the lifting surface lies on the free helical vortex sheets and has only a small deviation from this surface. This assumption is equivalent to the linearized theory of two-dimensional airfoils where the boundary condition is not satisfied on the profile, but on the profile chord.

Present subcavitating propeller lifting-surface procedure at NSRDC is mainly based on theoretical formulations by Pien (ref. 59) or Kerwin (ref. 60) for chordwise load effects on camber and pitch, together with the approach of Kerwin and Leopold (ref. 61) for the effects of blade thickness. The main feature of Pien's approach is that the numerical procedure is greatly simplified by subtracting out the lifting-line contribution, while Kerwin's method is based on the vortex lattice approach of Falkner (ref. 62). With regard to Pien's method, both Cheng (ref. 63) and Lerbs, Alef, and Albrecht (ref. 64) have formulated computation procedures for uniform chordwise loading, and Cheng (ref. 63) for NACA ($a=0.8$) type chordwise loading. NSRDC experience with the use of Cheng's procedure for NACA ($a=0.8$) chordwise loading, combined with Kerwin and Leopold's thickness procedure, indicates excellent comparisons with the results of model propeller tests. Various other theories and numerical approaches are due to Strecheletzky (ref. 65), Sparenberg (ref. 66), Yamazaki (ref. 67), Nelson (ref. 68), and Murray (ref. 69).

Lerbs, Alef, and Albrecht (ref. 64) present systematic lifting-surface calculation data for 48 propellers for a specific blade outline (nonskewed and skewed), optimum radial load distribution, constant chordwise load distribution, and varying number of blades, expanded area ratio, and hydrodynamic advance ratio. These results do not include blade thickness effects. More recently, Morgan, Silovic, and Denny (ref. 70) have performed a very comprehensive study of the Cheng/Kerwin and Leopold procedures and critically examined these procedures, upgraded output data by selective improvements in computational techniques, and presented a large amount of systematic lifting-surface data for a NACA ($a=0.8$) type chordwise load distribution, varying radial load, and blade thickness distributions, several blade outlines (nonskewed and skewed), varying number of blades, expanded area ratio, and hydrodynamic advance ratio. This latter data is particularly significant since NACA ($a=0.8$) type loading is realistic in real flow, whereas uniform chordwise loading is not, due to viscous effects. Results from these two sets of calculations, reinforced by earlier calculations of Pien, Cheng, Kerwin and Leopold, clearly demonstrate that

(1) The effect of blade skew is important and results in an ideal angle correction to the hydrodynamic pitch angle which is positive toward the root and negative toward the tip.

(2) The effect of blade thickness results in an ideal angle correction to the hydrodynamic pitch angle which is largest toward the root and diminishes toward the tip.

(3) Blade skew and thickness have a negligible effect on the chordwise distribution of blade camber.

(4) Chordwise loading which is symmetric about midchord does not result in any ideal angle correction to the hydrodynamic pitch angle.

(5) The concept of a camber correction factor is generally valid for practical design purposes. Moreover, with the use of complete lifting-surface procedures, it is feasible to define ideal angle correction factors for the effect of blade skew, thickness, and nonsymmetric loading about midchord.

Finally, and following the last conclusion, it should be mentioned that prior to the availability of complete lifting-surface procedures, finite aspect ratio effects had to be allowed for by the use of camber correction factors; i.e., a correction to the two-dimensional camber line. This type of correction factor was obtained by a limited application of lifting-surface theory which was formulated first in 1944 by Ludwig and Ginzler (ref. 71), who had to use graphical means to obtain a limited set of results. Later Cox (ref. 14) refined their theoretical approach and produced a more comprehensive set of correction factors using a digital computer. However, these factors were only determined for constant chordwise loading and left to semiempirical methods the problem of obtaining any necessary corrections to hydrodynamic pitch angle. The recent correction factors for camber and pitch derived by Morgan, Silovic, and Denny, which were also extracted from their complete lifting-surface calculations for the realistic NACA ($a=0.8$) type chordwise loading, will prove useful to the designer for several purposes; i.e., checking calculations, showing trends, back analysis, and cavitation calculations to account for operation in circumferentially varying inflow (refs. 52 and 72).

CONCLUSIONS

Based on the authors' combined experience and knowledge, it can be concluded that subcavitating propeller theoretical design procedures are generally adequate and in an advanced state of development. Present-day numerical procedures are sufficient to permit great flexibility to the designer in his choice of design parameters. Moreover, the consequences of design parameter choice are well understood. Future areas for investigation will probably tend to "off-design" performance rather than "design-point" performance.

It is concluded that the state-of-the-art for supercavitating propeller design is several years behind that for subcavitating propellers. Although

existing theory is well developed, much remains to be done in the area of numerical procedures; in addition, relatively little is understood regarding choice of design parameters for supercavitating propellers. Present-day design procedures for these propellers are semiempirical, but the way is open for a rapid upgrading in the next few years.

LIST OF SYMBOLS

A_E Expanded area ratio,

$$Z \int_{r_h}^R c \, dr$$

A_0 Disc area, πR^2

c Chordlength

C_c A strength coefficient which is dependent on chordwise camber distribution

C_{L_i} Section design lift coefficient,

$$\frac{L_i}{(\rho/2) c V_r^2}$$

C_P Propeller power coefficient,

$$\frac{P}{(\rho/2) \pi R^2 V_A^3} \quad \text{or} \quad \frac{P}{(\rho/2) \pi R^2 V^3}$$

C_s A strength coefficient which is dependent on chordwise thickness distribution

C_{T_h} Propeller thrust loading coefficient,

$$\frac{T}{(\rho/2) \pi R^2 V_A^2} \quad \text{or} \quad \frac{T}{(\rho/2) \pi R^2 V^2}$$

D Propeller diameter

EAR Expanded area ratio, A_E/A_0

J Advance coefficient, V_A/nD or V/nD

M_{x_0} Bending moment about blade chord

n Revolutions per unit time

p Static pressure at a reference position

p_c Cavity pressure (or vapor pressure)

P Power

P/D Pitch to diameter ratio

P_D Delivered power to propeller(s)

P_E Effective power, RV

r Radial distance

r_h	Propeller hub radius
R	Ship resistance or propeller radius
R_s	Ratio of minimum to maximum stress during a cycle
t	Thrust deduction factor, $R = T(1-t)$; or blade section maximum thickness
t/c	Section thickness-to-chord ratio
T	Propeller thrust
U	Reference velocity
V	Ship speed
V_A	Speed of advance of propeller
V_r	Resultant velocity to the blade section
V_t	Tangential component of ship wake
V_x	Axial component of ship wake
w	Taylor wake fraction, $V_A = V(1-w)$
Z	Number of blades
α	Angle of attack
$\beta(r)$	Advance angle, $\tan^{-1} (V_A/2\pi nr)$
η_B	Propeller behind efficiency, TV_A/P_D
η_D	Propulsive efficiency, P_E/P_D
η_H	Hull efficiency, $(1-t)/(1-w)$
λ	Advance ratio, J/π
σ	Cavitation number,

$$\frac{p_\infty - p_c}{(\rho/2) U^2}$$

σ_c	Allowable blade mean stress
σ_f	Fatigue stress for a given number of cycles, single amplitude
σ_{\max}	Maximum stress for a given number of cycles

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DISCUSSION

J. D. VAN MANEN (Netherlands Ship Model Basin): This joint paper contains a nearly complete review of the application of theory to propeller design.

One critical remark may be made with respect to the determination of the diameter, rpm, or number of blades. It is not necessary to determine these principal parameters of the propeller from series charts. In my opinion, an extension of theoretical computations with reliable drag coefficient values for the selected profiles may render fruitful, and in many cases more accurate, information for the determination of the optimum principal parameters. This procedure of determining optimum values holds not only from a viewpoint of efficiency but even from an economic or acoustic point of view.

G. E. THOMAS (Admiralty Research Laboratory): The authors are to be commended for this comprehensive review paper on propeller design.

They quite rightly suggest that a future field of investigation should be concerned with off-design performance. Equally important, however, is the need to develop experimental techniques to determine the flow field around the propeller so that adequate comparisons can be made with predictions by lifting-surface theory. A weakness at present is that the lifting-surface model is assessed on the basis of comparisons between experimental values and integrated values of thrust and torque at the design point. Until the design method is validated by satisfactory correlation between the pressure distribution as measured over the blade and that predicted by theory, is it strictly correct to refer to the $a=0.8$ meanline as being more "realistic"?

The loss of lift due to viscous effects on two-dimensional airfoils (of the order of 5 to 20 percent) has been satisfactorily explained by many authors by using the concept of displacement thickness to allow for boundary-layer growth. It is presumed that no such procedure has been incorporated in the authors' design method. It would be appreciated, therefore, if more clarification could be given by the authors as to the basis for their statement that the $a=0.8$ meanline is "more realistic."

COX AND MORGAN (authors): We wish to thank the discussors for their comments. Professor Van Manen stated that he preferred to use theoretical computation methods in preference to propeller series charts

for preliminary design purposes. We are in complete agreement with this approach and regret our lack of emphasis on this point in the paper.

Mr. Thomas commented on the authors' viewpoint regarding the advantage of using NACA ($a=0.8$) type chordwise loading as opposed to a uniform chordwise distribution. All the evidence of which we are aware for two-dimensional airfoil and propeller model tests indicates that chordwise loading of the NACA ($a=0.8$) type meets its theoretical pressure distribution reasonably well, whereas NACA ($a=1.0$) type loading most certainly does not. Of course, it is not maintained that $a=0.8$ loading is necessarily the very best choice, especially for rotating flows. We are aware that a number of investigators have attempted to use the concept of displacement thickness to allow for boundary-layer growth and loss of lift due to viscous effects. However, we are not aware that this procedure has yet been successful. We prefer not to incorporate unproven yet complex techniques into a computer program when a judicious selection of the meanline pressure distribution alleviates the difficulty.

Mr. Thomas is quite correct in stating that good agreement for total thrust and torque values does not properly indicate the adequacy of the theoretical design procedures. In addition to comparing the total thrust and torque values, we always examined the cavitation patterns for the propellers we have designed. These results indicate that the radial load distribution is reasonable. We agree that this approach only gives an indication; hence, our engineers have developed techniques for measuring the pressure distribution on a propeller (ref. D-1). Pressure distributions will be obtained soon for a propeller designed by lifting-surface theory.

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